

A See-saw model of sterile neutrino

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Abstract

If the smallness of the mass of the sterile neutrino is to be explained by the see-saw mechanism, the off-diagonal entries of the mass matrix needs to be protected by some symmetry not far above the electroweak scale. We implement see-saw mechanism in a gauge model based on $SU(2)_L^q \times SU(2)_L^l \times U(1)_Y^q \times U(1)_Y^l$ un-unified gauge group which breaks to $SU(2)_L \times U(1)_Y$ at the TeV region via a two-step symmetry breaking chain. The right handed diagonal block is tied to the highest scale up to which the un-unification symmetry holds. The sterile neutrino emerges from a quark-lepton mixed representation of the un-unified group.

A light sterile neutrino (ν_s) is necessary to explain the solar[1], atmospheric[2] and LSND[3] neutrino anomalies simultaneously via neutrino mixing schemes. The sterile neutrino has to be approximately degenerate in mass with either ν_e or ν_μ and it must have a mixing angle compatible with those found by the oscillation experiments. Neutrinos in the eV mass range can also play the role of hot dark matter[4] of the universe. If solar neutrino anomaly is explained through the matter-induced oscillation $\nu_e \leftrightarrow \nu_s$ the corresponding mixing angle $\sin^2 2\Theta_{es}$ can either be 10^{-2} or of the order of unity with $\Delta m_{es}^2 \sim 10^{-5} \text{ eV}^2$. In this case the atmospheric neutrino anomaly has to be explained through maximal $\nu_\mu \leftrightarrow \nu_\tau$ mixing. On the contrary if solar neutrino anomaly is explained through $\nu_e \leftrightarrow \nu_\tau$ oscillation, the atmospheric neutrino anomaly has to be explained through maximal $\nu_\mu \leftrightarrow \nu_s$ oscillation with $\Delta m_{\mu s}^2 \sim 10^{-3} \text{ eV}^2$. The invisible decay width of the Z boson work oneself into a position that there are three neutrinos[5] coupling to the $SU(2)_L \times U(1)$ invariant weak neutral current. Fourth neutrino has to be a singlet under the standard model gauge symmetry, or in other words, it has to be sterile.

A natural way to get a small neutrino mass is via the see-saw mechanism[6]. In this case the mass matrix can formally be written as,

$$\begin{matrix} & \nu_L & \nu_R \\ \nu_L & \begin{pmatrix} 0 & m_D \\ m_D & M_X \end{pmatrix} \end{matrix}. \quad (1)$$

Note that the off-diagonal Dirac type mass is $G \equiv SU(2)_L \times U(1)_Y$ symmetry breaking. This makes the off diagonal entry of the order of m_Z . The diagonal entry, however, is G conserving and can be taken to be a large scale of the order of the right-handed symmetry breaking scale (M_R) or the GUT scale (M_X). We have generically termed this scale M_X later. The matrix has two eigenvalues m_D^2/M_X and M_X . The first eigenvalue explains the smallness of the neutrino mass when $M_X \rightarrow \infty$. If on the other hand the off-diagonal entry is also G conserving, the mass eigenvalues will be of the order of $M_X^2/M_X = M_X$ and M_X . Obviously in this case see-saw mechanism cannot explain the smallness of neutrino mass. This situation arises in the case of a singlet or sterile neutrino as in this case the off-diagonal elements originate from singlet Higgs scalars. In this paper our purpose is to study a minimally modified version of the standard model gauge symmetry G and construct a model for the sterile neutrino in such a way that a naturally

light sterile neutrino as well as the required mixing angles with the ordinary species can be explained through see-saw mechanism.

As we stick to the see-saw mechanism, the Dirac type off-diagonal entry of the sterile neutrino have to be protected near the electroweak scale by some symmetry[7]. In this paper we will consider a variation on an ingenious gauge model based on quark-lepton un-unification symmetry [8, 10]

$$G^0 = SU(q)_L^q \times SU(2)_L^l \times U(1)_Y, \quad (2)$$

which breaks to standard model gauge group at a scale $M_E \sim O(TeV)$. Our gauge group was studied in Ref([11]) and we will see that it will nicely fits to our purpose. The breaking chain is,

$$\begin{aligned} G' &= SU(2)_L^q \times SU(2)_L^l \times U(1)_Y^q \times U(1)_Y^l, \\ M_X \longrightarrow G^0 &= SU(2)_L^q \times SU(2)_L^l \times U(1)_Y \\ M_E \longrightarrow G &= SU(2)_L \times U(1)_Y. \end{aligned} \quad (3)$$

When the electric charge is expressed in terms of the generators of G' , we get

$$Q = T_q^3 + T_l^3 + Y^q + Y^l. \quad (4)$$

The global fits of all electroweak precision parameters put limit on the mass scales of the extra gauge bosons belonging to group G^0 . The model based on G^0 has been studied in literature extensively. In Ref[12] it has been shown that the additional gauge bosons should be heavier than 2 TeV depending on the new mixing angle of the un-unified group defined similar to the weak mixing angle of the standard model. The heavier gauge bosons will induce additional box diagrams contributing to $B_0 - \overline{B}_0$ mixing. Furthermore the deviation of $e^+ e^- \rightarrow \mu^+ \mu^-$ and $e^+ e^- \rightarrow b \overline{b}$ asymmetries from the standard model predictions restrict the mixing of ordinary and extra gauge bosons [13, 14, 15]. In this letter, we will simply choose the VEV of $H_E = 2$ TeV as a tentative value inspired by [12] varying the Yukawa couplings. We could have taken a different value of $\langle H_E \rangle$ which will yeald a separate set of Yukawa couplings. Quark mass generation in this model is complicated. An approach is sketched in Ref[8].

We add extra fermions $S_L = (2, 2, 1/2, -1/2)$ and their right handed singlet counterparts S_R . S_L contains a singlet of G which will be the left-handed partner of our sterile neutrino. The right handed sterile neutrino is

a singlet of G'^1 . Furthermore a Higgs scalar field $(2,2,1/2,-1/2)$ is needed to break the group G' . This is the lowest dimensional non-trivial representation which contains a singlet of G .

The model based on group G' is theoretically incomplete. The standard model particle content introduces triangle anomalies as separate anomalies related to the quark and leptonic parts do not add up to zero individually. A prescription of additional fermions in similar models is forwarded in References[8, 15]. We will give the details of extra fermions which cancel anomaly and their effects on quark masses and flavor changing neutral currents in along the lines of [11] in a future publication. Let us summarize the particle content. The fermions and scalars transform in the notation $G' \rightarrow G$ as,

FERMIONS

$$\begin{aligned} Q_L &= (2, 1, 1/6, 0) \rightarrow (2, 1/6) \\ U_R &= (1, 1, 2/3, 0) \rightarrow (1, 2/3) \\ D_R &= (1, 1, -1/3, 0) \rightarrow (1, -1/3) \\ L_L &= (1, 2, 0, -1/2) \rightarrow (2, -1/2) \\ E_R &= (1, 1, 0, 1) \rightarrow (1, 1) \end{aligned}$$

EXTRA FERMIONS

$$\begin{aligned} N_R &= (1, 1, 0, 0) \rightarrow (1, 0) \\ S_L &= (2, 2, 1/2, -1/2) \rightarrow (3, 0) + (1, 0) \\ S_R &= (1, 1, 0, 0) \rightarrow (1, 0) \end{aligned}$$

HIGGSES

$$\begin{aligned} H &= (1, 2, 0, 1/2) \rightarrow (2, 1/2) \\ H_E &= (2, 2, 0, 0) \rightarrow \langle H_E \rangle \sim M_E \end{aligned}$$

¹One can consider an even more baroque but symmetric scheme such as SU(16) [9] where the right handed sterile neutrino also emerges from the bi-doublet representation

$$H_X = (1, 1, 1/2, -1/2) \rightarrow \langle H_X \rangle \sim M_X$$

Note that the bi-doublet has a triplet and a singlet at low energy. The singlet is interpreted as the left handed sterile neutrino. Given the particle spectrum it is easy to construct the neutrino mass matrix in terms of the VEVs of the fields,

$$\langle H_X \rangle = \eta, \quad \langle H_E \rangle = \sigma, \quad \langle H \rangle = v. \quad (5)$$

For this toy model it is enough to consider only the electron generation. We will check whether a $\nu_e \leftrightarrow \nu_s$ solution of the solar neutrino problem is possible. The Atmospheric and LSND solutions will depend on the Yukawa matrix of the three active neutrino species². The simplified neutrino mass matrix is

$$\begin{array}{c} \nu_L \quad S_L \quad N_R \quad S_R \\ \begin{array}{c} \nu_L \\ S_L \\ N_R \\ S_R \end{array} \begin{pmatrix} 0 & 0 & h_1 v & h_2 v \\ 0 & 0 & h_3 \sigma \frac{\eta}{M_X} & h_4 \sigma \frac{\eta}{M_X} \\ h_1 v & h_3 \sigma \frac{\eta}{M_X} & O(M_X) & O(M_X) \\ h_2 v & h_4 \sigma \frac{\eta}{M_X} & O(M_X) & O(M_X) \end{pmatrix} \end{array} \quad (6)$$

The non-renormalizable Yukawa couplings h_3 and h_4 are interesting. They keep the effective strength of the off-diagonal elements at the TeV region. The light neutrino masses are the two light eigenvalues of Eqn.(6). It is easy to see that they are also the eigenvalues of the light neutrino mass matrix given by

$$m_{light} = m_{Dirac} \frac{1}{M} m_{Dirac}^\dagger \quad (7)$$

where, symbolically we have expressed m_{dirac} as the off-diagonal 2×2 block and M as bottom-right 2×2 block of the matrix in Eqn.(6). We will further assume

$$M = M_X \begin{pmatrix} h_5 & h_6 \\ h_7 & h_8 \end{pmatrix} \quad (8)$$

Now we are in a position to give the results. We take parameters as,

$$v = 256 \text{ GeV}, \quad \sigma = 2000 \text{ GeV}, \quad M_X = 10^{16} \text{ GeV} \quad (9)$$

and then try to find natural values of Yukawa couplings in the range $10^{-3} - 1$ which gives $\Delta m^2 \sim 10^{-5}$ and $\sin^2 2\Theta \sim 1 - 2\%$. Note that $\langle \sigma \rangle = 2 \text{ TeV}$

²We have checked that it is also possible to get the $\nu_s \leftrightarrow \nu_\mu$ solution to the atmospheric neutrino anomaly in a similar manner.

is a choice value and should only be looked upon a sample value in the TeV range. An exact value of $\langle \sigma \rangle$ is not required for our purposes. A set of solution is given in Table (1). It's not the precise values in table (1) that are the "results", but the concept they represent (one can avoid fine-tuning while providing sterile neutrinos).

h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	$\sin^2 2\Theta$	Δm^2
0.1	0.05	0.05	0.1	1.0	0.4	0.4	1.0	0.022	$10^{-5.1}$
0.1	0.04	0.04	0.1	1.0	0.4	0.4	1.0	0.010	$10^{-5.1}$
0.1	0.04	0.04	0.1	1.0	0.3	0.3	1.0	0.016	$10^{-5.2}$

Table 1: A set of natural values of neutrino Yukawa couplings giving desired masses and mixing

We note that in the standard model the $U(1)_Y$ symmetry protects the lepton masses from shooting up to the Plank scale as all representations of the $SU(2)$ group are self-conjugate. Had we considered the group $G^0 = SU(2)_L^q \times SU(2)_L^l \times U(1)_Y$ instead of G' and introduced the extra fermion $S_L = (2, 2, 0)$ under G^0 we would have had the same consequence. The $S_L S_L$ entry of the mass matrix in Eqn (6) wouldn't have been protected around the TeV scale. This is the justification of using G' in this paper.

To conclude, the standard model is constructed in such a way that the neutrino remains massless. If we want to have a neutrino mass in the eV range from the VEV of the standard model Higgs doublet we need to add a right handed neutrino and the corresponding 'Dirac type' neutrino Yukawa coupling needs to be fine tuned to the precision of $10^{-9}/10^2 = 10^{-11}$. A natural solution to this is the see-saw mechanism where the mass of the light neutrino emerges as m_{weak}^2/M_X from the diagonalization of a 'Majorana-type' mass matrix. We obtain a light neutrino in the eV range when $M_X \sim 10^{13.8}$ GeV, where M_R is some large scale of the theory. However the sterile neutrino, being a singlet, does not feel the effect electroweak symmetry breaking as it does not couple to the Higgs doublet which breaks the electroweak symmetry. All mass scales relevant to the sterile neutrino shoots off to the largest scale M_X up to which the standard model symmetry remains exact. Thus the see-saw mechanism breaks down. This is because see-saw mechanism necessarily

needs an interplay between two scales, the weak scale and the scale M_X in the present circumstances.

In this note we have constructed a scenario where the quark lepton unification symmetry exists near the TeV scale and this symmetry gives the required protection to the Dirac type off-diagonal mass of a sterile neutrino, which is embedded in a bi-doublet representation of the un-unified group. Hence in this scenario the sterile neutrino is a low energy manifestation of a quark-lepton ‘bi-doublet’ mixed representation which feels the effect of the breaking of the un-unified symmetry. We assume that the VEV σ which breaks the un-unification group is at the TeV range whereas the quark-lepton U(1) groups ($U(1)^l$ and $U(1)^q$) merge at the high scale M_X . We have used *natural values* of all the Yukawa couplings ($0.001 - 1.0$) and succeeded in obtaining the feasible mass scale of the sterile neutrino and its mixing with the electron neutrino in the context of the solar neutrino anomaly. Thus, in doing so we did not need to fine-tune the Yukawa couplings. This may explain the solar neutrino anomaly. In doing this we have used a variation of the see-saw mechanism.

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References

- [1] N. Hata and P. Langacker, Phys. Rev. **D56**, 6107 (1997) and contained references
- [2] K. S. Hirata et al, Phys. Lett. **B205**, 416 (1988); Phys. Lett. **B280**, 146 (1992); R. Becker-Szendy et. al, Phys. Rev. **D46**, 3120 (1992); M. Goodman et. al., Nucl. Phys. **B38**. (Proc. Supp.), 337 (1995). Y. Suzuki, Talk presented at Neutrino 98, Takayama, Japan, June 1998.
- [3] C. Athanassopoulos et al, Phys. Rev. Lett. **81** 1774 (1998); nucl-ex/9706006

- [4] Q. Shafi and F. W. Stecker, Phys. Rev. Lett **53**, 1292 (1984); J. R. Primack, J. Holtzman, A. Klypin and D. O. Caldwell, Phys. Rev. Lett. **74**, 2160 (1995). J. Primack, *Proceedings of XVII International Conference in Neutrino Physics and Astrophysics*, Neutrino 96, Helsinki, Finland, eds K. Enqvist, K. Huitu and J. Maalampi, World Scientific, Singapore 1997.
- [5] See for example, Edward Gerard Hefflin (UC, Riverside). UMI-94-14489-mc (microfiche), Dec 1993. Ph.D.Thesis.
- [6] M. Gell-Mann, P. Rammond and R. Slansky, in Supergravity, eds D. Freedman et al, North Holland Publishing Company, Amsterdam, 1980; T. Yanagida, in Proceedings of the KEK workshop, 1979 (unpublished). R. N. Mohapatra and G. Senjanović. Phys. Rev. Lett **44**, 912 (1980).
- [7] $[SU(2)_L \times U(1)_Y]_{Mirror}$ is used in Z. G. Berezhiani, R. N. Mohapatra, Phys. Rev. **D52** 6607 (1995); B. Brahmachari, R. N. Mohapatra, Phys. Lett. **B437**, 100 (1998)
- [8] H. Georgi, E. E. Jenkins and E. H. Simmons, Phys. Rev. Lett. **62** 2789,(1989); Erratum-ibid. **63** 1540, (1989); Nucl. Phys.B **331** 541 (1990).
- [9] B. Brahmachari, Phys. Rev. **D48**, 1266 (1993).
- [10] E. Ma and S. Rajpoot, Mod. Phys. Lett. **A5** 979 (1990); Erratum-ibid. **A5** 1529 (1990)
- [11] D. Choudhury, Mod. Phys. Lett. **A6** 1185 (1991)
- [12] R.S. Chivukula, E.H. Simmons and J. Terning, Phys. Lett. **B346** 284 (1995)
- [13] V. Barger and T. Rizzo, Phys. Rev. **D41**, 946,(1990); A. Donini, F. Feruglio, J. Matias and F. Zwirner, Nucl. Phys. **B507** 51 (1997).
- [14] L. Randall, Phys. Lett. **B234** 508 (1990)
- [15] T. G. Rizzo, Int. J. Mod. Phys. **A7** 91 (1992)